case and may not be representative of other cases.

The wave equations for E and H pertaining to inhomogeneous media are:

$$\nabla \times \nabla \times E - k_0^2\epsilon_r E = 0 \tag{1}$$

$$\nabla \times \nabla \times H - \nabla \ln \epsilon_r \times \nabla \times H - k_0^2\epsilon_r H = 0 \tag{2}$$

where E and H are the electric and magnetic field components, $k_0^2 = \omega^2\epsilon_0\mu_0$ and ϵ_r is the relative dielectric constant, assumed a function of the radial distance only.

As is usual, the following functional forms are assumed for E and H:

$$\begin{aligned} E_r &= f_1(r)\cos n\phi & H_r &= g_1(r)\sin n\phi \\ E_\phi &= f_2(r)\sin n\phi & H_\phi &= g_2(r)\cos n\phi \\ E_z &= f_3(r)\cos n\phi & H_3 &= g_3(r)\sin n\phi. \end{aligned} \tag{3}$$

A factor $\exp j(\omega t - k_z z)$ is suppressed. In terms of f and g, (1) and (2) reduce to the following set of coupled equations:

$$\frac{\partial^2 f_1}{\partial r^2} + \left(\frac{1}{r} + \frac{\partial \ln \epsilon_r}{\partial r}\right)\frac{\partial f_1}{\partial r} + \left(k_0^2\epsilon_r + \frac{\partial^2 \ln \epsilon_r}{\partial r^2} - \frac{1+n^2}{r^2} - k_z^2\right) f_1 = \frac{2n}{r^2} f_2 \tag{4}$$

$$\frac{\partial^2 f_2}{\partial r^2} + \frac{1}{r}\frac{\partial f_2}{\partial r} + \left(k_0^2\epsilon_r - \frac{1+n^2}{r^2} - k_z^2\right) f_2 = n\left(\frac{2}{r^2} + \frac{1}{r}\frac{\partial \ln \epsilon_r}{\partial r}\right) f_1 \tag{5}$$

$$\frac{\partial^2 f_3}{\partial r^2} + \frac{1}{r}\frac{\partial f_3}{\partial r} + \left(k_0^2\epsilon_r - \frac{n^2}{r^2} - k_z^2\right) f_3 = jk_z \frac{\partial \ln \epsilon_r}{\partial r} f_1 \tag{6}$$

$$\frac{\partial^2 g_1}{\partial r^2} + \frac{1}{r}\frac{\partial g_1}{\partial r} + \left(k_0^2\epsilon_r - \frac{1+n^2}{r^2} - k_z^2\right) g_1 = -\frac{2n}{r^2} g_2 \tag{7}$$

$$\frac{\partial^2 g_2}{\partial r^2} + \left(\frac{1}{r} - \frac{\partial \ln \epsilon_r}{\partial r}\right)\frac{\partial g_2}{\partial r} + \left(k_0^2\epsilon_r - \frac{1+n^2}{r^2} - k_z^2 - \frac{1}{r}\frac{\partial \ln \epsilon_r}{\partial r}\right) g_2 = -n\left(\frac{2}{r^2} + \frac{1}{r}\frac{\partial \ln \epsilon_r}{\partial r}\right) g_1 \tag{8}$$

$$\frac{\partial^2 g_3}{\partial r^2} + \left(\frac{1}{r} - \frac{\partial \ln \epsilon_r}{\partial r}\right)\frac{\partial g_3}{\partial r} + \left(k_0^2\epsilon_r - \frac{n^2}{r^2} - k_z^2\right) g_3 = jk_z \frac{\partial \ln \epsilon_r}{\partial r} g_1. \tag{9}$$

These equations become uncoupled when

$$\text{a) } \frac{\partial}{\partial \phi} = 0; \quad \text{or} \quad n = 0;$$

$$\text{b) } \epsilon_r = \frac{l}{r^2} \tag{10}$$

where l is a constant. We consider case b when (5) reduces to:

$$\frac{\partial^2 f_2}{\partial r^2} + \frac{1}{r}\frac{\partial f_2}{\partial r} + \left(\frac{k_0^2 l - 1 - n^2}{r^2} - k_z^2\right) f_2 = 0. \tag{11}$$

Its solution is

$$f_2(r) = AI_m(k_z r) + BK_m(k_z r) \tag{12}$$

where I_m and K_m are the modified Bessel functions of order m and $m^2 = 1 + n^2 - k_0^2 l$. A and B are constants. Their ratio is determined by the boundary conditions $f_2 = 0$ at $r = a, b$, say:

$$-\frac{B}{A} = \frac{I_m(k_z a)}{K_m(k_z a)} = \frac{I_m(k_z b)}{K_m(k_z b)}. \tag{13}$$

This equation in k_z has solutions for imaginary m only, hence the cutoff condition:

$$k_z^2 > \frac{1+n^2}{}. \tag{14}$$

The equation determines the possible values of k_z also. Similarly (8) reduces to:

$$\frac{\partial^2 g_2}{\partial r^2} + \frac{3}{r}\frac{\partial g_2}{\partial r} + \left(\frac{k_0^2 l + 1 - n^2}{r^2} - k_z^2\right) g_2 = 0. \tag{15}$$

Its solution is:

$$g_2(r) = \frac{1}{r}\left[A' I_p(k_z r) + B' K_p(k_z r)\right] \tag{16}$$

where A' and B' are constants, and $p^2 = n^2 - k_0^2 l$.

All other field components can be determined in terms of $f_2(r)$ and $g_2(r)$, for example

$$f_3(r) = -\frac{j}{\alpha}\left[r^2 \frac{\partial g_2}{\partial r} + r g_2 + r\beta f_2\right]$$

$$\alpha = \frac{k_0^2 l - n^2}{\omega\mu}; \qquad \beta = \frac{nk_z}{\omega\mu}. \tag{17}$$

On applying boundary conditions to f_3 we find

$$-\frac{B'}{A'} = \frac{I_p'(k_z a)}{K_p'(k_z a)} = \frac{I_p'(k_z b)}{K_p'(k_z b)} \tag{18}$$

where I' and K' are the derivatives with respect to r.

It will be observed that (13) and (18) represent different conditions on k_z. If both these equations are not satisfied by the same value of k_z, the solution must be regarded as a TE wave with finite $f_2\ g_1\ g_3$ and a TM wave with $g_2\ f_1\ f_3$. These solutions do not satisfy the divergence equation unless $n = 0$. They represent different modes with different propagation constants.

For some values of the parameters a, b, l, (13) and (18) are satisfied simultaneously. In this special case solutions exist for $n \neq 0$ also. The divergence equation $\nabla \cdot D = 0$ serving to reduce the number of arbitrary constants $A\ B\ A'\ B'$ to one. The solution represents EH type modes.

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Dielectric Constant of Atlas at 9363 MHz

Monoarseniate double d'ammonium de thallium (Atlas), $(NH_4)_4Tl_3(H_2AsO_4)_7$, has been reported to be ferroelectric. The complex dielectric constant of Atlas has been measured by LeDonche [1]. These measurements made at 50 Hz displayed ferroelectric hysteresis loops in the temperature range between 150°K and 110°K with maximum dielectric constant occurring at 150°K. At room temperature Atlas is hexagonal.[1] The present correspondence reports measurements of the temperature dependence of the dielectric constant at 9363 MHz at right angles to the "c" axis of the crystal, i.e., along the length of the crystal.

The complex dielectric constant was measured by the cavity perturbation [2] method. For a rod placed parallel to, and at the maximum value of the electric field, one can write [2]

$$\left(\frac{\delta\omega}{\omega}\right) - j\delta\left(\frac{1}{2Q}\right) = (\epsilon' - 1 - j\epsilon'')\int_{\Delta V} E^2 dv/2\int_V E^2 dv \tag{1}$$

where ω is the angular resonant frequency, Q is the quality factor, and V is the volume of the cavity. E is the field inside the empty cavity and is assumed to be unaffected by the introduction of the specimens of volume ΔV. The variations $\delta\omega$ and $\delta(1/2Q)$ represent shifts due to the insertion of the specimen. The real part of the relative dielectric constant is designated ϵ', the imaginary part ϵ''. Similar expressions are given by Champlin and Krongard [3].

For the dielectric material placed at the center of a rectangular waveguide resonating in the TE_{10n} mode one has [4]

$$\left(\frac{\delta\omega}{\omega}\right) = -(\epsilon' - 1)\frac{2\Delta V}{V} \tag{2}$$

and

$$\frac{1}{Q_e} = \epsilon''\frac{4\Delta Y}{V} \tag{3}$$

$$\frac{1}{Q_L} = \frac{1}{Q_e} + \frac{1}{Q_u} \tag{4}$$

where Q_e, Q_u, and Q_L are the quality factors of the dielectric material, the unloaded and the loaded cavity, respectively.

The essential parts of the experimental set-up are shown in Fig. 1. The klystron was swept by the sawtooth voltage through the cavity resonance which was displayed on CRO 2. The klystron output, over the mode, was displayed on CRO 1. The time sweep of both the oscilloscopes were carefully adjusted to be the same. The horizontal scale of CRO 1 was calibrated in frequency with the wavemeter and this calibration was used to measure the 3 dB points of the cavity resonance on CRO 2.

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[1] X-ray measurements were made by W. Carter at Prof. F. Kanda's Laboratory at Syracuse University.

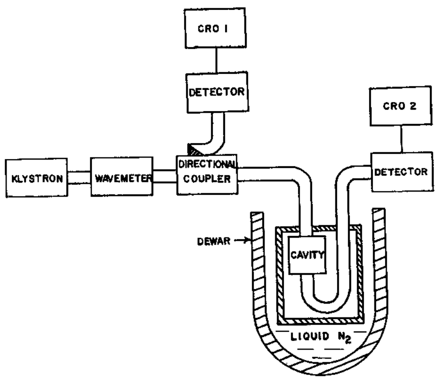

Fig. 1. Microwave dielectric constant measuring apparatus.

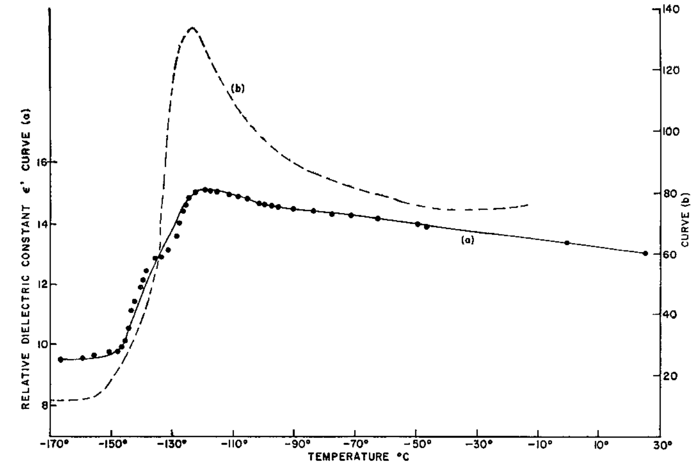

Fig. 2. Relative dielectric constant of Atlas (a) at 9363 MHz for fields perpendicular to the "*c*" axis, and (b) as measured by LeDonche [1].

A thermocouple was connected on the cavity close to the specimen. The cavity was enclosed in an aluminum can. The specimen was cooled down to liquid N_2 temperature and was allowed to warm up slowly by the natural evaporation of liquid N_2. A temperature run was made with the sample in place and then an empty cavity run was made.

A cavity was constructed from standard rectangular waveguide and operated in the TE_{103} mode. A small hole was provided at the center of the broad walls of the cavity to facilitate the removal of the rod without opening the cavity. The rods[2] were approximately 8.5 mm by 1.5 mm by 0.8 mm. As the rod cross section was rather irregular, the mean volume was obtained from measurements of weight; the density of Atlas being taken[3] as 4.83 g/cc. The rod was supported in a thin polyfoam cylinder. Results of several runs on various samples are shown in Fig. 2(a). The peak value of the relative dielectric constant occurs at -121 ± 1°C, close to that obtained by LeDonche [1]. Results obtained by LeDonche are plotted in Fig. 2(b). As the orientation of the crystal, for LeDonche's measurements, is not known the values of the dielectric constant obtained by the author and LeDonche could not be directly compared. The room temperature (26°C) value of the reciprocal loss tangent $1/\tan\delta$ is 75. This is slightly lower than that obtained by LeDonche at 50 Hz.

Ferroelectricity in Atlas has been shown by LeDonche [1] by measuring both the temperature and the electric field dependence of the dielectric constant at 50 Hz. The temperature dependence of the dielectric constant, similar to that observed by LeDonche, at microwave frequencies indicates that the ferroelectricity in Atlas is maintained at these high frequencies. Since the loss tangent is fairly low, the nonlinear properties of Atlas could be profitably utilized for building microwave devices.

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[2] The Atlas rods were obtained due to the courtesy of L. LeDonche, Faculte des Sciences, Rennes, France.

[3] Measurements were made by W. Carter at Prof. F. Kanda's Laboratory at Syracuse University.

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